Mathématiques discrètes et applications
Discrete Mathematics and Applications

Problem Session 1

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Exercise 1-a

NB: Two statement forms are logically equivalent if and only if they have identical truth values for all possible combinations of truth values of their statement variables.

$\neg(p \vee q) \stackrel{?}{\equiv} \neg p \wedge \neg q$

p	q	$\neg p$	$\neg q$	$p \vee q$	$\neg(p \vee q)$	$\neg p \wedge \neg q$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	T

· The last two columns have the same truth values for all possible combinations of truth values of p and q; so we can conclude that $\neg(p \vee q)$ and $\neg p \wedge \neg q$ are equivalent.

Exercise 1-a

$\neg(p \wedge q) \stackrel{?}{\equiv} \neg p \vee \neg q$

p	q	$\neg p$	$\neg q$	$p \wedge q$	$\neg(p \wedge q)$	$\neg p \vee \neg q$
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

Exercise 1-b

$$p \wedge (q \vee r) \stackrel{?}{\equiv} (p \wedge q) \vee (p \wedge r)$$

p	q	r	$(p \wedge q)$	$(p \wedge r)$	$(q \vee r)$	$p \wedge (q \vee r)$	$(p \wedge q) \vee (p \wedge r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	F	F	F	F
F	T	T	F	F	T	F	F
F	T	F	F	F	T	F	F
F	F	T	F	F	T	F	F
F	F	F	F	F	F	F	F

Exercise 1-c

NB: A statement form is a tautology if it is true for all possible combinations of truth values of its statement variables.

$p \to (p \vee q) \stackrel{?}{\equiv} \mathbf{T}$

p	q	$(p \vee q)$	$p \to (p \vee q)$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

Exercise 1-d

$p \oplus (q \oplus r) \stackrel{?}{\equiv} (p \oplus q) \oplus r$

p	q	r	$(p \oplus q)$	$(q \oplus r)$	$p \oplus (q \oplus r)$	$(p \oplus q) \oplus r$
T	T	T	F	F	T	T
T	T	F	F	T	F	F
T	F	T	T	T	F	F
T	F	F	T	F	T	T
F	T	T	T	F	F	F
F	T	F	T	T	T	T
F	F	T	F	T	T	T
F	F	F	F	F	F	F

Exercise 2-a

$$\overbrace{\neg(\neg p \wedge q) \wedge (p \vee q)}^{\text{L.H.S}} \stackrel{?}{\equiv} \overbrace{p}^{\text{R.H.S}}$$

$$\begin{aligned}
L.H.S &= \neg(\neg p \wedge q) \wedge (p \vee q) && \\
&\equiv (\neg\neg p \vee \neg q) \wedge (p \vee q) && \textbf{De Morgan's law} \\
&\equiv (p \vee \neg q) \wedge (p \vee q) && \textbf{Double negative law} \\
&\equiv p \vee (\neg q \wedge q) && \textbf{Distributive law} \\
&\equiv p \vee (q \wedge \neg q) && \textbf{Commutative law} \\
&\equiv p \vee \mathbf{F} && \textbf{Negation law} \\
&\equiv p && \textbf{Identity law} \\
&= R.H.S \quad \square &&
\end{aligned}$$

Exercise 2-b

$$\overset{\text{L.H.S}}{a \rightarrow b} \overset{?}{\equiv} \overset{\text{R.H.S}}{\neg a \vee b}$$

a	b	$\neg a$	$\neg a \vee b$	$a \rightarrow b$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

Let $E : a \rightarrow b \equiv \neg a \vee b$

Exercise 2-b

$$\overbrace{(p \to r) \wedge (q \to r)}^{\text{L.H.S}} \stackrel{?}{\equiv} \overbrace{(p \vee q) \to r}^{\text{R.H.S}}$$

$$\begin{aligned}
L.H.S &= (p \to r) \wedge (q \to r) & \\
&\equiv (\neg p \vee r) \wedge (\neg q \vee r) & \textbf{Follows from E} \\
&\equiv (r \vee \neg p) \wedge (r \vee \neg q) & \textbf{Commutativity of } \vee \\
&\equiv r \vee (\neg p \wedge \neg q) & \textbf{Distributive law} \\
&\equiv r \vee \neg(p \vee q) & \textbf{De Morgan's law} \\
&\equiv \neg(p \vee q) \vee r & \textbf{Commutavity of } \vee \\
&\equiv (p \vee q) \to r & \textbf{E} \\
&= R.H.S \quad \square &
\end{aligned}$$

Exercise 2-c

$$\overbrace{(p \wedge q) \to r}^{\text{L.H.S}} \overset{?}{\equiv} \overbrace{p \to (q \to r)}^{\text{R.H.S}}$$

$R.H.S = p \to (q \to r)$	
$\equiv \neg p \vee (\neg q \vee r)$	**E**
$\equiv (\neg p \vee \neg q) \vee r$	**Associative law**
$\equiv \neg(p \wedge q) \vee r$	**De Morgan's law**
$\equiv (p \wedge q) \to r$	**E**
$= L.H.S \quad \square$	

Exercise 2-d

$$\overset{\text{L.H.S}}{p \leftrightarrow q} \overset{?}{\equiv} \overset{\text{R.H.S}}{(p \land q) \lor (\neg p \land \neg q)}$$

$$\begin{aligned}
R.H.S &= (p \land q) \lor (\neg p \land \neg q) && \\
&\equiv ((p \land q) \lor \neg p) \land ((p \land q) \lor \neg q) && \textbf{Distrib. law} \\
&\equiv (\neg p \lor (p \land q)) \land (\neg q \lor (p \land q)) && \textbf{Comm. law} \\
&\equiv ((\neg p \lor p) \land (\neg p \lor q)) \land ((\neg q \lor p) \land (\neg q \lor q)) && \textbf{Distrib. law} \\
&\equiv (T \land (\neg p \lor q)) \land ((\neg q \lor p) \land T) && \textbf{Negation law} \\
&\equiv (\neg p \lor q) \land (\neg q \lor p) && \textbf{Identity law} \\
&\equiv (p \to q) \land (q \to p) && \textbf{E} \\
&\equiv p \leftrightarrow q && \textbf{Def. of } \leftrightarrow \\
&= L.H.S \quad \square &&
\end{aligned}$$

Exercise 2-e

$$\overset{\text{L.H.S}}{p \wedge (p \vee q)} \overset{?}{\equiv} \overset{\text{R.H.S}}{p}$$

$$\begin{aligned}
L.H.S &= p \wedge (p \vee q) && \\
&\equiv (p \vee \mathbf{F}) \wedge (p \vee q) && \textbf{Identity law} \\
&\equiv p \vee (\mathbf{F} \wedge q) && \textbf{Distributive law} \\
&\equiv p \vee (q \wedge \mathbf{F}) && \textbf{Commutative law} \\
&\equiv p \vee \mathbf{F} && \textbf{Universal bound law} \\
&\equiv p && \textbf{Identity law} \\
&= R.H.S \quad \square &&
\end{aligned}$$

Exercise 2-e

$$\overbrace{p \vee (p \wedge q)}^{\text{L.H.S}} \stackrel{?}{\equiv} \overbrace{p}^{\text{R.H.S}}$$

$$\begin{aligned}
L.H.S &= p \vee (p \wedge q) & \\
&\equiv (p \wedge \mathbf{T}) \vee (p \wedge q) & \textbf{Identity law} \\
&\equiv p \wedge (\mathbf{T} \vee q) & \textbf{Distributive law} \\
&\equiv p \wedge (q \vee \mathbf{T}) & \textbf{Commutative law} \\
&\equiv p \wedge \mathbf{T} & \textbf{Universal bound law} \\
&\equiv p & \textbf{Identity law} \\
&= R.H.S \quad \square &
\end{aligned}$$

Exercise 3: Important concepts

· "**if**" introduces a **sufficient**, *but not necessary* condition. For ex., "P if Q" means $Q \rightarrow P$. I.e. Q is a sufficient, but not necessary condition for P.

· "**only if**" introduces a **necessary**, *but not sufficient* condition. For ex., "P only if Q" means $P \rightarrow Q$. I.e. Q is necessary, but not sufficient condition for P.

· "**if and only if**" (iff) is the conjunction of "if" and "only if" and introduces a **necessary and sufficient condition**. For ex., "P if and only if Q" means "$Q \rightarrow P$ and $P \rightarrow Q$" $\equiv P \leftrightarrow Q$. I.e. Q is a necessary and sufficient condition for P (and vice versa).

Exercise 3

Consider the following statements:

p : "You will be hired"

q : "You major in mathematics or computer science, get a B avg. or better, and take accounting".

The director's statement can be represented as: $p \rightarrow q$.

Consider the truth table of $p \rightarrow q$:

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

If you major in mathematics or computer science, get a B avg. or better, take accounting, and do not get hired, this corresponds to the third row in the truth table: i.e. $p = F$ and $q = T$. From the truth table, the director's statement, $p \rightarrow q$, remains true.

Exercise 3

Conclusion: The director did not lie to you.

In other words, majoring in mathematics or computer science, getting a B avg. or better, and taking accounting is a **necessary, but not sufficient** condition to get hired.

Exercise 4

Consider the following statements:

p: "Compound X is boiling."
q: "The temperature of X is at least 150°C."

The logical content of the statement in the exercise is: $p \to q$.

(a) $q \to p$ (Converse) $\not\equiv p \to q$: False. In fact, if p is false, but q is true, then $p \to q$ is true, but $q \to p$ is false.

(b) $\neg q \to \neg p$ (Contrapositive) $\equiv p \to q$: True.

(c) p only if $q \equiv p \to q$: True.

(d) $\neg p \to \neg q$ (Inverse) $\not\equiv p \to q$: False. In fact, if p is false, but q is true, then $p \to q$ is true, but $\neg p \to \neg q$ is false.

End

For more exercises and notes, see Rosen 7th Edition Chap. 1