University of Neuchâtel
Discrete Mathematics and Applications - Fall 2025
Problems - 1

Part A: Logic proofs

1. Proofs with truth tables

   (a) De Morgan's laws for two variables are: $\neg(p \vee q) \equiv \neg p \wedge \neg q$ and $\neg(p \wedge q) \equiv \neg p \vee \neg q$. Using truth tables, prove the correctness of these laws.

   (b) Show the distributivity of the $\wedge$ operator over the $\vee$ operator.

   (c) Using a truth table, prove that $p \to (p \vee q)$ is a tautology.

   (d) Prove the associativity of the $\oplus$ operator.

2. Proofs without truth tables

   (a) Show that: $\neg(\neg p \wedge q) \wedge (p \vee q) \equiv p$

   (b) Show that $(p \to r) \wedge (q \to r)$ and $(p \vee q) \to r$ are logically equivalent, using $a \to b \equiv \neg a \vee b$.

   (c) Show that: $(p \wedge q) \to r \equiv p \to (q \to r)$

   (d) Prove that: $p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$

   (e) Prove the correctness of the absorption laws.

Part B: Worded problems

3. Thinking about your academic future, you go to the company **UniSphere** to ask what you should do during your studies in order to be hired after graduation. The company's CEO explains that you will be recruited **only if** you choose mathematics or computer science as your major, maintain at least a B average, and take a Web Programming course. So you decide to major in mathematics, you achieve a B+ average, and you take the Web Programming course. Once you graduate, you return to UniSphere, you formally apply ... but your application is rejected.

   Did the CEO lie to you?

4. "If compound X is boiling, then its temperature must be at least 150°C." Assuming that this statement is true, which of the following must also be true?

   (a) If the temperature of compound X is at least 150°C, then compound X is boiling.

   (b) If the temperature of compound X is less than 150°C, then compound X is not boiling.

   (c) Compound X will boil only if its temperature is at least 150°C.

   (d) If compound X is not boiling, then its temperature is less than 150°C.